Physics 371 Spring 2017 Prof. Anlage
Review

Special Relativity

Inertial vs. non-inertial reference frames

Galilean relativity: Galilean transformation for relative motion along the $x - x'$ direction:

$x = Vt + x'$ and the inverse transformation: $x' = x - Vt$

$y = y'$ $\quad$ $y' = y$

$z = z'$ $\quad$ $z' = z$

$t = t'$ $\quad$ $t' = t$

$\vec{v}' = \vec{v} - \vec{V}$.

Michelson-Morley experiment – no evidence for an ether that acted as the medium for the propagation of light.

Einstein's postulates:

1) If S is an inertial reference frame and if a second frame S' moves with constant velocity relative to S, then S' is also an inertial reference frame.
2) The speed of light (in vacuum) has the same value *c* in every direction in all inertial reference frames.

Time dilation: $\Delta t = \gamma \Delta t'$, where $\gamma = 1/\sqrt{1-\beta^2}$, and $\beta = V/c$.

Length contraction: $\ell = \ell_0/\gamma$, where ℓ_0 is the 'proper length' of an object.

Lorentz transformation for relative motion along the $x - x'$ direction: $x' = \gamma(x - Vt)$, $y' = y$, $z' = z$, $t' = \gamma(t - xV/c^2)$, and the inverse transformation: $x = \gamma(x' + Vt')$, $y = y'$, $z = z'$, $t = \gamma(t' + x'V/c^2)$.

Spacetime Four-vectors

$$x^{(4)} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}, x^{(4)} = (\vec{x}, ct),$$

The Lorentz transformation as a rotation in 4-dimensional spacetime:

$$x'^{(4)} = \overline{\overline{\Lambda}}\, x^{(4)}, \overline{\overline{\Lambda}} = \begin{pmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{pmatrix}$$

Invariant length of a four-vector: $s \equiv {x_1}^2 + {x_2}^2 + {x_3}^2 - {x_4}^2$,

$$x^{(4)} \cdot y^{(4)} \equiv x_1 y_1 + x_2 y_2 + x_3 y_3 - x_4 y_4$$

Relativistic velocity transformation:

$v'_x = \frac{v_x - V}{1 - Vv_x/c^2}$, $v'_y = \frac{v_y}{\gamma(1 - Vv_x/c^2)}$, $v'_z = \frac{v_z}{\gamma(1 - Vv_x/c^2)}$, and the inverse: $v_x = \frac{v'_x + V}{1 + Vv'_x/c^2}$, $v_y = \frac{v'_y}{\gamma(1 + Vv'_x/c^2)}$, $v_z = \frac{v'_z}{\gamma(1 + Vv'_x/c^2)}$

The Relativistic Doppler effect

$f_{Approach} = f_0\sqrt{\frac{1+\beta}{1-\beta}}$, $f_{Receed} = f_0\sqrt{\frac{1-\beta}{1+\beta}}$.

The light cone; absolute future and absolute past.

Velocity four-vector: $u^{(4)} = \frac{dx^{(4)}}{dt_0} = \gamma(u)(\vec{u}, c)$,
Relativistic momentum: $p^{(4)} = m\gamma(u)(\vec{u}, c) = (m\gamma(u)\vec{u}, E/c)$,
Relativistic energy: $E = \gamma(u)mc^2$, Three-momentum: $\vec{p} = m\gamma(u)\vec{u}$,
Kinetic energy: $T \equiv E - mc^2 = (\gamma(u) - 1)mc^2$,
$\vec{\beta} = \frac{\vec{u}}{c} = \vec{p}c/E$, $\vec{u} = \frac{\vec{p}}{\gamma(u)m} = \vec{p}c^2/E$.
The useful relation: $p^{(4)} \cdot p^{(4)} = -(mc)^2$, $E^2 = (\vec{p}c)^2 + (mc^2)^2$,
Relativistic four-momentum: $p^{(4)} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ E/c \end{pmatrix}$,
Lorentz transformation of four-momentum: $p'^{(4)} = \bar{\bar{\Lambda}}\, p^{(4)}$
Photon: $E = pc = \hbar\omega$, $\omega = kc$, $p = \frac{\hbar\omega}{c} = \hbar k$, $p_\gamma^{(4)} = \hbar\left(\vec{k}, \frac{\omega}{c}\right) = \frac{\hbar\omega}{c}(\hat{k}, 1)$, $p_\gamma^{(4)} \cdot p_\gamma^{(4)} = 0$,
Compton scattering: $\lambda - \lambda_0 = \frac{h}{mc}(1 - \cos\theta)$, $\lambda_C = \frac{hc}{mc^2} = 2.43\, pm$ (for electron scattering)
Relativistic four-force: $K^{(4)} = \frac{dp^{(4)}}{dt_0} = \gamma\left(\frac{d\vec{p}}{dt}, \frac{1}{c}\frac{dE}{dt}\right) = \gamma\left(\vec{F}, \frac{1}{c}\vec{u} \cdot \vec{F}\right)$

Thermodynamics

Thermodynamic variables. Extensive vs. Intensive quantities
Thermodynamic functions of state (P, V, T, U, S, thermodynamic potentials) have exact differentials
An equation of state relates thermodynamic functions of state
Ideal gas: $PV = Nk_BT$, where $k_B = 1.38 \times 10^{-23} J/K$ is Boltzmann's constant. $U = \frac{3}{2}Nk_BT$.
Heat is thermal energy in transit. Heat is <u>not</u> a thermodynamic state function
Heat capacity $C = dQ/dT$, specific heat $c = C/M$. $C_V = dQ/dT|_V$, $C_P = dQ/dT|_P$, $\frac{C_P}{C_V} = \gamma$ the adiabatic index.
Heat and work are <u>not</u> thermodynamic state functions.
Inexact differentials $ðW = -PdV$, $ðQ = TdS$.
Work done on a gas $ðW = -PdV$.
Zeroth Law of Thermodynamics – Thermal equilibrium
First law of thermodynamics $\Delta U = \Delta W + \Delta Q$, or $dU = ðW + ðQ$.
Reversible vs. irreversible processes
Isothermal, adiabatic, isochoric, isobaric processes
Heat engine: A machine that produces work from a temperature difference between two reservoirs
Carnot cycle (isothermal and adiabatic processes)
Carnot efficiency: $\eta_{Carnot} = \frac{W}{Q_{Hot}} = 1 - \frac{T_{Cold}}{T_{Hot}}$.
Carnot cycle: $\frac{Q_{Hot}}{T_{Hot}} = \frac{|Q_{Cold}|}{T_{Cold}}$, and $\oint \frac{ðQ_{rev}}{T} = 0$, leading to $dS = \frac{ðQ_{rev}}{T}$. 2nd Law of Thermodynamics: $\Delta S_{Universe} \geq 0$.

Kelvin statement of the 2nd law: No process is possible whose sole result is the complete conversion of heat into work.
Clausius statement of the 2nd law: No process is possible whose sole result is the transfer of heat from a colder to a hotter system.
Otto cycle – internal combustion engine
Refrigerator

Re-statement of the 1st-law: $dU = TdS - PdV$.
Thermodynamic potentials: Enthalpy: $H = U + PV$; $dH = TdS + VdP$. Helmholtz Function: $F = U - TS$; $dF = -pdV - SdT$. Gibbs Function: $G = U + PV - TS$; $dG = -SdT + VdP$.

Constraints	**Minimized Potential**
Fixed V, T	Helmholtz Function $F = U - TS$
Thermally isolated, Fixed V	Entropy S
Fixed T, P	Gibbs Function $G = U + PV - TS$
Thermally isolated, fixed P	Enthalpy $H = U + PV$

Phase transitions: Latent Heat: $L = T_{boil}\left(S_{steam} - S_{liquid}\right)$. The order of a phase transition is the order of the lowest differential of G that shows a discontinuity at T_c
van der Waals equation of state: $[P + (N^2/V^2)a][V - Nb] = Nk_BT$. Accounts for excluded volume and attractive forces between molecules. $P_c = \frac{a}{27b^2}$; $V_c = 3Nb$; $T_c = \frac{8a}{27b}$, so $\left[\frac{P}{P_c} + 3\left(\frac{V_c}{V}\right)^2\right]\left[\frac{V}{V_c} - \frac{1}{2}\right] = \frac{8}{3}\frac{T}{T_c}$, the "law of corresponding states"
Isothermal compressibility $\kappa_T \equiv -\frac{1}{V}\frac{\partial V}{\partial P}|_T$. $\frac{\partial V}{\partial P}|_T = \frac{1}{\frac{\partial P}{\partial V}|_T}$. Maxwell construction to define the vapor/liquid co-existence region: $G_{vapor} - G_{liquid} = 0 = \int_{Liq}^{vap} dG = \int_{Liq}^{vap} VdP$.

Quantum Physics

JJ Thomson charge-to-mass measurement in E, B fields: $q/m = \frac{v}{RB}$. Millikan oil droplet experiment: revealed the quantization of electric charge.
Blackbody radiation, Stefan-Boltzmann law: $R_{Total} = \sigma T^4$, $\sigma = 5.6703 \times 10^{-8}\frac{W}{m^2K^4}$. Wien displacement law says that $\lambda_{max}T = 2.898 \times 10^{-3} m - K$.
Radiation power per unit area related to the energy density of a blackbody: $R(\lambda) = \frac{c}{4}\rho(\lambda)$.
Rayleigh-Jeans (classical equipartition argument) law $\rho(\lambda) = 8\pi k_BT/\lambda^4$ leads to the 'ultraviolet catastrophe'.
Planck blackbody radiation (treat the atoms as having discrete energy states, and the light as having energy $E = hf$): $\rho(\lambda) = \frac{8\pi hc/\lambda^5}{e^{hc/\lambda k_BT} - 1}$, $h = 6.626 \times 10^{-34} J - s$.
Photoelectric effect and the concept of light as a particle (photon with $E = hf$): $hf = eV_0 + \phi$. Photon collides with one electron and transfers all of its energy, $-V_0$ is the stopping potential.

X-ray production by Bremsstrahlung with cutoff $\lambda_{min} = \frac{1240}{V}\ nm$ (Duane-Hunt Rule), explained by Einstein as inverse photoemission with $\lambda_{min} = \frac{hc}{eV}$. Sharp emission lines arise from quantized energy levels in the 'core shells' of atoms.
Bragg reflection of x-rays from layers of atoms in crystals: $n\lambda = 2d\sin\theta$, where $n = 1, 2, 3, \ldots$, d is the spacing between the parallel layers.
Rutherford scattering (Phys 410) suggested that positive charge is concentrated in a very small volume – the nuclear model of the atom.
Empirical rule for light emission from hydrogen $\frac{1}{\lambda_{mn}} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$, Rydberg constant $R = R_H = 1.096776\ \times 10^7\ \frac{1}{m}$ for Hydrogen.
Bohr model of the hydrogen atom (assumes stationary states, light comes from transitions between stationary states, electron angular momentum in circular orbits is quantized): $\left|\vec{L}\right| = |\vec{r} \times m\vec{v}| = mvr = n\hbar$, with $n = 1, 2, 3, \ldots$, Radius of circular orbits: $r_n = \frac{n^2 a_0}{Z}$ with $a_0 = \frac{4\pi\varepsilon_0\hbar^2}{me^2} = 0.529$ Å, Total energy of Hydrogen atom: $E_n = -E_0\frac{Z^2}{n^2}$, with $E_0 = \frac{mc^2\left(e^2/4\pi\varepsilon_0\right)^2}{2\,(\hbar c)^2} = \frac{mc^2}{2}\alpha^2 = 13.6\ eV$, $\alpha = \frac{e^2/4\pi\varepsilon_0}{\hbar c} \cong \frac{1}{137}$ is called the 'fine structure constant'. Explains the Hydrogen atom emission spectrum but not multi-electron atoms.
Davisson-Germer experiment shows that matter (electrons) diffract from periodic structures (Ni atoms on a surface) like waves. It is clear that matter has a strong wave-like character when measured under appropriate conditions.
deBroglie proposed the wavelength of matter waves as $\lambda_{dB} = h/p$, where p is the linear momentum. Classical physics should be recovered in the short-λ_{dB} limit – the Correspondence Principle
The time-dependent Schrodinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$;
Separation of variables leads to $\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$ (a property of stationary states);
Time-independent Schrodinger equation: $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\,\psi(x) = E\psi(x)$;
The wavefunction $\Psi(x,t)$ is complex in general and cannot be measured. Born interpretation of the wave function in terms of a probability density $P(x,t) = \Psi^*(x,t)\Psi(x,t)$;
Normalization condition: $\int_{-\infty}^{+\infty}|\psi(x)|^2\,dx = 1$.
Particle of mass m in an infinite square well between $x = 0$ and $x = L$: $E_n = \frac{\hbar^2 {k_n}^2}{2m} = n^2\frac{\pi^2\hbar^2}{2mL^2}$ with $n = 1, 2, 3, \ldots$, and $\psi_n(x) = \sqrt{2/L}\ \sin k_n x$.
Finite square well of height V_0, energy eigenvalues are solutions of the transcendental equation: $\tan\left(\frac{\sqrt{2mE}}{\hbar}a\right) = \sqrt{\frac{V_0 - E}{E}}$ (even parity solutions). Always at least one solution!
Harmonic oscillator: $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + \frac{1}{2}m\omega^2x^2\psi(x) = E\psi(x)$, $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$, where $n = 0, 1, 2, 3, \ldots$, Eigenfunctions: $\psi_n(x) = C_n\,e^{-m\omega^2x^2/2\hbar}\,H_n(x)$, involve the Hermite polynomials multiplying a Gaussian in x.
Classical turning points are inflection points in $\psi(x)$.
General wave uncertainty properties: $(\Delta x)\ (\Delta k) \geq 1/2$, $(\Delta t)\ (\Delta\omega) \geq 1/2$.

Quantum uncertainty properties: $(\Delta x)\ (\Delta p) \geq \hbar/2$, $(\Delta t)\ (\Delta E) \geq \hbar/2$.
Expectation values: $\langle x \rangle = \int_{-\infty}^{\infty} \Psi^*(x,t)\ x\ \Psi(x,t)\ dx$, and for any function of position: $\langle f(x) \rangle = \int_{-\infty}^{\infty} \Psi^*(x,t)\ f(x)\ \Psi(x,t)\ dx$

Linear momentum operator: $p_{op} = -i\hbar \frac{\partial}{\partial x}$, Hamiltonian operator: $\mathcal{H}_{op} = \frac{{p_{op}}^2}{2m} + V(x)$, the time independent Schrodinger equation written as an operator equation: $\mathcal{H}_{op}\psi(x) = E\ \psi(x)$.

Step potential $V(x) = \begin{cases} 0 & \text{for } x < 0 \\ V_0 & \text{for } x > 0 \end{cases}$ has reflection rate $R = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2$, and transmission rate $T = \frac{4\ k_1\ k_2}{(k_1 + k_2)^2}$, where $k_1 = \sqrt{2mE}/\hbar$ and $k_2 = \sqrt{2m(E - V_0)}/\hbar$.

Tunneling rate through a barrier $T = \left[1 + \frac{\sinh^2(\alpha a)}{4\frac{E}{V_0}\left(1 - \frac{E}{V_0}\right)}\right]^{-1} \approx 16\ \frac{E}{V_0}\left(1 - \frac{E}{V_0}\right) e^{-2\alpha a}$, where a is the barrier width, and $\alpha = \sqrt{2m(V_0 - E)}/\hbar$.